Calculating Galactic Distances with Cepheid Variables

Edwin Hubble (1889-1953) was the first astronomer to calculate the distance to the Andromeda galaxy. His results ultimately showed that the universe was expanding. Hubble determined the distance based on data he collected from observing Cepheid variables in Andromeda. A Cepheid variable is a pulsating star, that expands and contracts on a regular basis over a given period of time. This regular expansion and contraction results in the star's luminosity changing accordingly. For example, the Cepheid variable Eta Aquilae in figure 1 has a period of 7-days. Its apparent magnitude ranges between 3.5 and 4.3 over a 7-day period.

Day	Apparent magnitude
1	**3.6**
2	**3.9**
3	**4.2**
4	**4.3**
5	**4**
6	**3.7**
7	**3.5**
8	**3.8**

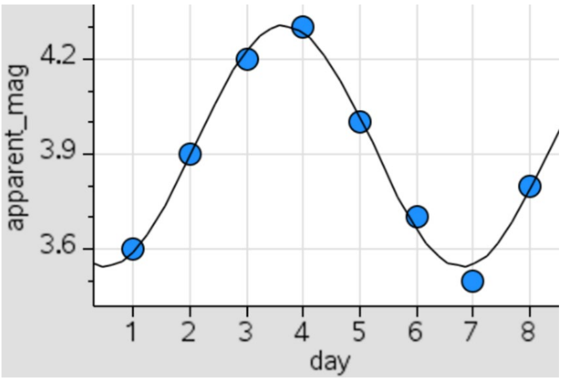

Figure 1: Plot of apparent magnitude verses days for the Cepheid variable Eta Aquilae

The study of Cepheids and the relationship between their periods and luminosities were first determined by Henrietta Leavitt in 1912. Leavitt worked as a "computer" at the Harvard College Observatory examining photographic plates of 1777 stars in the Small and Large Magellanic Clouds. After classifying 47 as Cepheids, she examined the relationship between the period and the brightness of 25 selected Cepheids. Leavitt found that the longer the period, the brighter the star was on the photographic plate. Since these 25 stars were all approximately the same distance away in the Magellanic Clouds, identifying a Cepheid with a distinct period in another galaxy would allow for a distance estimate, based on its apparent magnitude. Cepheids were in fact "standard candles" which could be used for measuring distances across the void of space. Leavitt plotted her 25 Cepheids and found that there was a distinct relationship between the log of the periods of Cepheids and the apparent magnitudes (m) of the Cepheids. This is illustrated in the table of values and the graph in figure 2 below.

Cepheid	Apparent Magnitude	Period (days)
1	11.45	31.43
2	11.24	35.58
3	11.84	31.21
4	11.41	34.23
5	11.17	37.46
6	11.63	32.92
7	11.38	36.97
8	11.21	34.01
9	11.79	30.59
10	11.32	35.76
11	11.51	32.11
12	11.14	37.65
13	11.27	35.01
14	11.72	31.87
15	11.36	36.21
16	11.43	34.98
17	11.78	30.88
18	11.29	35.32
19	11.19	37.12
20	11.66	32.43
21	11.49	33.76
22	11.26	35.43
23	11.97	29.94
24	11.33	34.76
25	11.15	36.56

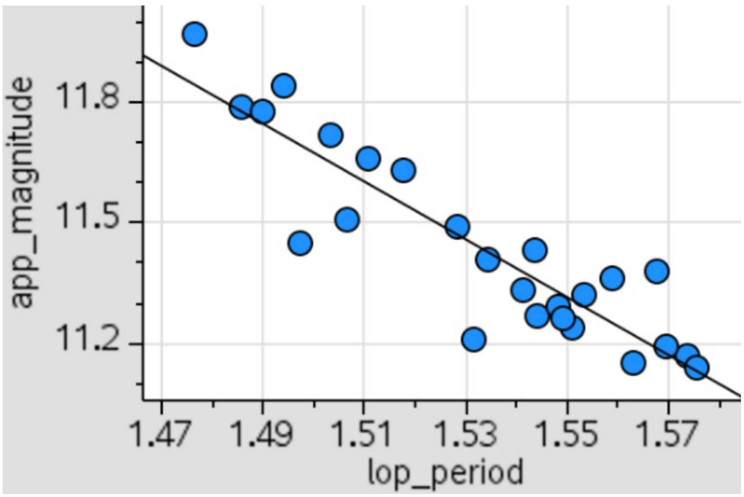

Figure 2: The table lists the apparent magnitude and period of 25 Cepheid variables. Leavitt plotted the log of the period against the apparent magnitude to obtain a linear relationship. She found that brighter Cepheids had longer periods.

Leavitt's equation relating apparent magnitude (m) and period (P) was then converted into an equation involving absolute magnitude (M) and period (P).

$$M = -2.76\log(P) - 1.4$$

The relationship became known as the Period Luminosity relationship or Leavitt's law, whereby the absolute magnitude (M) of a Cepheid is linked to the log of its period (in days).

For example, suppose a Cepheid in a distant galaxy has a period of 30 days, then its absolute magnitude (M) is:

$$M = -2.76\log(30) - 1.4$$
$$= -5.48$$

To calculate the distance to a galaxy, all that is needed is the apparent magnitude (m) and the period of a Cepheid in the galaxy of interest. The distance formula is then used to determine the distance in parsecs. The distance formula relates the apparent magnitude (m), absolute magnitude (M) and the distance d (in parsecs) as follows:

$$m - M = 5\log(d) - 5$$

One parsec is approximately 3.26 light years and the absolute magnitude (M) of a star is its magnitude when viewed from a distance of 10 parsecs or 32.6 light years.
The equation can be rearranged to give the following:

$$d = 10^{(m-M+5)/5}$$

The expression $m - M$ is known as the distance modulus and is a measure of the distance to the object. If the distance modulus is 0, then the object is exactly 10 parsecs away. If the distance modulus is negative, the object is closer than 10 parsecs and its apparent magnitude(m) is brighter than its absolute magnitude (M). If the distance modulus is positive, the object is further than 10 parsecs and its apparent magnitude (m) is less bright than its absolute magnitude (M).
To determine the distance to a galaxy, several Cepheids are observed so that the apparent magnitude and period of each Cepheid can be calculated. For example, the light curve below in figure 3 is for a Cepheid in the Large Magellanic Clouds.

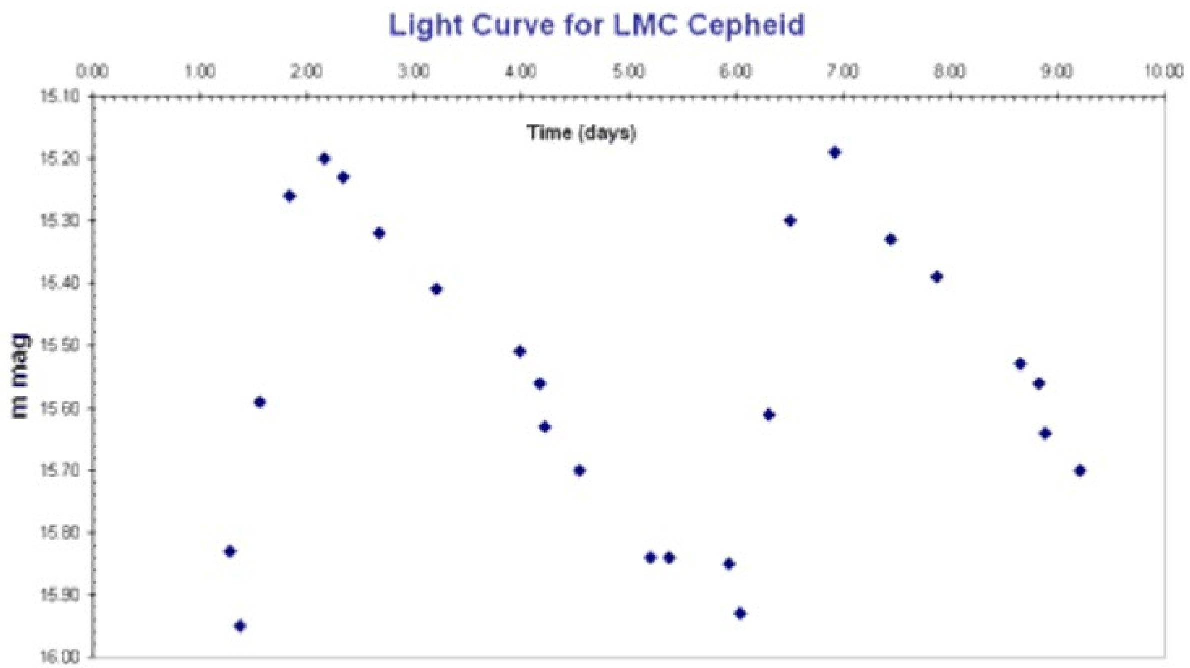

Figure 3: Light curve for a Cepheid in the Large Magellanic Clouds. (Source: National Telescope Facility)

From the light curve, the apparent mean magnitude and the period are all that are required to determine the distance to the Large Magellanic Clouds. (In reality several Cepheids are monitored for precise results)

Apparent mean magnitude (m) = 15.56 and the period = 4.76 days.
The absolute magnitude (M) is first determined by using the equation:

$M = -2.76\log(P) - 1.4$
$M = -2.76\log(4.76) - 1.4$
$= -3.27$

These apparent magnitude (m), and absolute magnitude (M) are then subbed into the distance equation.

$d = 10^{(m-M+5)/5}$
$d = 10^{(15.56-(-3.27)+5)/5}$
$d = 10^{4.766}$
$= 58344.5\ parsecs$ or 190203 light years

Questions

1. A Cepheid variable has a period of 18 days and an apparent magnitude of 14.3. How far away is the Cepheid variable star.

Solution:
First determine absolute magnitude (M)
$M = -2.76\log(18) - 1.4$
$= -4.86$
Next substitute the apparent magnitude (m) and absolute magnitude (M) into the distance formula.
$d = 10^{(m-M+5)/5}$
$= 10^{(14.3-(-4.86)+5)/5}$
$= 10^{4.832}$
$= 67920.4$ parsecs or 221420.4 light years.

2. A Cepheid variable has a period of 10 days and an apparent magnitude of 12.5. Calculate its distance from Earth using the period-luminosity relation and the distance modulus formula.

Solution:
$M = -2.76\log(10) - 1.4$
= -4.16
$d = 10^{(m-M+5)/5}$
$= 10^{(12.5-(-4.16)+5)/5}$
$= 10^{4.332}$
$= 21478.3$ parsecs or 70019.3 light years.

Leavitt's ground breaking work was ignored for over four years until Ejnar Hertzsprung recognised the implications of what the unsung Leavitt had done. Edwin Hubble suggested that she should receive the Nobel Prize, while the Swedish mathematician Gosta Mittag-Leffler went even further and wrote her a letter in 1926 about nominating her for the Nobel Prize, not realizing that she had been dead for 4 years. As the Nobel Prize is not awarded posthumously, Henerietta never received her nomination.

References

Miss Leavitt's Stars, George Johnson, Atlas Books, W.W. Norton & Co., 2005

Australian Telescope Facility
https://www.atnf.csiro.au/outreach/education/senior/astrophysics/variable_cepheids.html#: